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PLANNING THE IMPLEMENTATION OF CONSTRUCTION SCHEDULES WITH CONSIDERATION OF TIME BUFFERS

Abstract

This paper contains a description of the assumptions of time buffers application in construction schedules. The Goldratt method referring to the critical chain method and the application of time buffers are presented. In the paper, the Authors have proposed how time buffers should be modified to include the specific character of large construction schedules. An original concept of new types of buffers and their location in the relationship network is presented. Additionally, various techniques of sizing time buffers depending on the accepted assumptions have been proposed. The theoretical part is concluded by the presentation of an algorithm worked out by the authors which determines the enterprise completion date including time buffers. The second part presents a full calculation example of a schedule for modernization of sewage works. Construction of the CPM schedule, as well as methods of locating and sizing of the buffers proposed by the authors are presented. Four variants of enterprise completion are determined depending on the assumed shortened task duration and type of distribution of the completion date.

Keywords*: CPM schedule, time project buffers, Goldratt's method, critical path, project delay risk management*

1 Introduction

Network methods of planning civil structures have been developing since several tens of years, i.e. from the elaboration of the CPM method. In the beginning they were based on a network defined at the edges (known also as ADM models – Arrow Diagramming Methods) and were related to time analysis in a deterministic sense. In 1958 another method developed, i.e. the PERT (Program Evaluation and Review Technique), which, based on the same assumptions for a network of relationships defined at the edges, allowed time analysis in a probabilistic sense [Połoński 2011]. This was the beginning of a stochastic approach to enterprise accomplishment. Developments of these basic methods are e.g. GERT (Graphical Evaluation and Review Technique), GERTS (Graphical Evaluation and Review Technique Simulation) and CYCLONE (CYCLic Operations NEtwork) [Jaworski 1999]. Since that time many new methods have appeared that are more precise, easier in application and digitized [Kapliński 2008], based on PDM (Precedence Diagramming Method) definition of the relationship network.

Practice shows that construction schedules often become invalid. This causes increase of the enterprise schedule time or its particular stages. There are many reasons of this state. They are linked both with mishaps (such as e.g. weather conditions, equipment failure, untimely material supply, etc.), as well as imprecise planning methods and estimating the particular task duration times. Methods are sought that will allow a risk of changes at the planning stage, which may develop during the accomplishment of the accepted plan. One of such concepts is the critical chain method.

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2 Assumptions of the critical chain method and application of time buffers

The founder of the Critical Chain Method (known also as the Critical Chain Construction Project – CCPM or Critical Chain Scheduling and Buffer Management CC/BM) is E. Goldratt (1997). He proposed supplementing network methods with several additional elements known as the Theory of Constraints (TOC). The entire methodology serves to aid project management when the completion dates for particular tasks are unsure and is an extension of the CPM and PERT methods. It is most commonly used in computer science, rarely in civil engineering [Milian 2004a].

The CC/BM method aims at reducing the influence of "Parkinson's Law" and "student's syndrome". They indicate that regardless the real amount of work, task duration always engages a planned value (or even exceeds it), and the beginning of work is always delayed to the latest-start date. In his book, Goldratt expresses this issue as follows: "First work out a time reserve. When you obtain it, you have plenty of time, so why hurry? When do you start work? You postpone it to the latest possible moment. This is just human nature." [Goldratt 1997]).

In the Critical Chain Method reduction of the enterprise completion date is obtained by reducing particular task durations, thus removing individual reserves of particular tasks. It is assumed that the initial probability of fulfilling a single task duration is c. 90%. Such high probability of fulfilling the duration of subsequent tasks leads to significant increase of the enterprise duration – which is not always justified. However, it is not always reflected in the real duration of work in each task. Therefore, according to the critical chain concept, the task durations should be reduced to a certain value. Goldratt estimates this reduction at 30–50% of the initial duration (with initial probability of its fulfilling at 90%).

All reserves obtained with reduction of task durations are transferred to the time buffer, located at the end of the critical chain and referred to the project buffer (PB). In turn, the critical chain is defined as the 'longest set of dependent tasks necessary to obtain the project goal with optimal consideration of all restrictions" [Milian 2006]. In the case when resources are unlimited, this definition concurs with the definition of the critical chain in the CPM method. Change of the project buffer size allows to regulate the enterprise completion date and the probability of fulfilling this date.

Figure 1A presents an example of a sequence of tasks. Their durations are reflected in the proportional length of the rectangles. Connections between the tasks in the form of arrows reflect only the relation between the sequence elements. Duration times for particular tasks are given with 90% certainty of their fulfilment. The next figure (Figure 1B) presents the same task sequence, which according to the critical chain method was shortened to ensure fulfilment of its duration at the level of 50% certainty. The resulting project buffer attained its size due to transfer of the reduced durations of the critical tasks to this element. Its size at this moment is the sum of reductions of all tasks that it refers to. However, the enterprise duration in both cases is the same. In the last case (Figure 1C), the time float of the project buffers was changed. The method, as mentioned above, does not estimate the exact value of reduction, which depends on the specific character of each planned enterprise. The figure shows that in this case the project buffer was reduced by 50%, and the finish date was shortened by 25% [Milian 2004a].

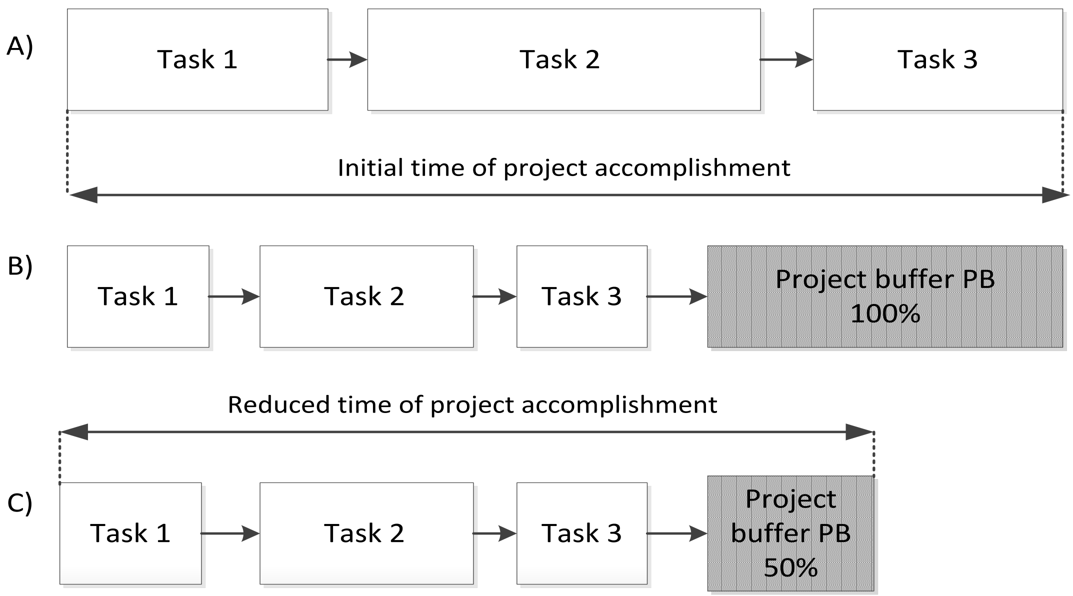

Figure 1. Scheme of project buffer (PB) location.
Source: Z. Milian

The CC/BM method assumes that only one critical chain can occur in the schedule and it cannot change during the modification of the task durations. However, reduction of task durations may lead to change of the critical chain course. This can be prevented by the introduction of feeding buffers (FB) by Goldratt. They are located at the end of non-critical sequences connected with the critical chain (Figure 2) and focused on protecting the start date in the critical chain with which the feeding buffer is connected (Figure 2a), as well as preventing the critical chain against changing its course [Milian 2004b]. In the case when the supply sequence is connected with the network at the end of the enterprise, a feeding buffer should be located at the end of the non-critical path before the project buffer (Figure 2b), otherwise the course of the critical chain would be changed.

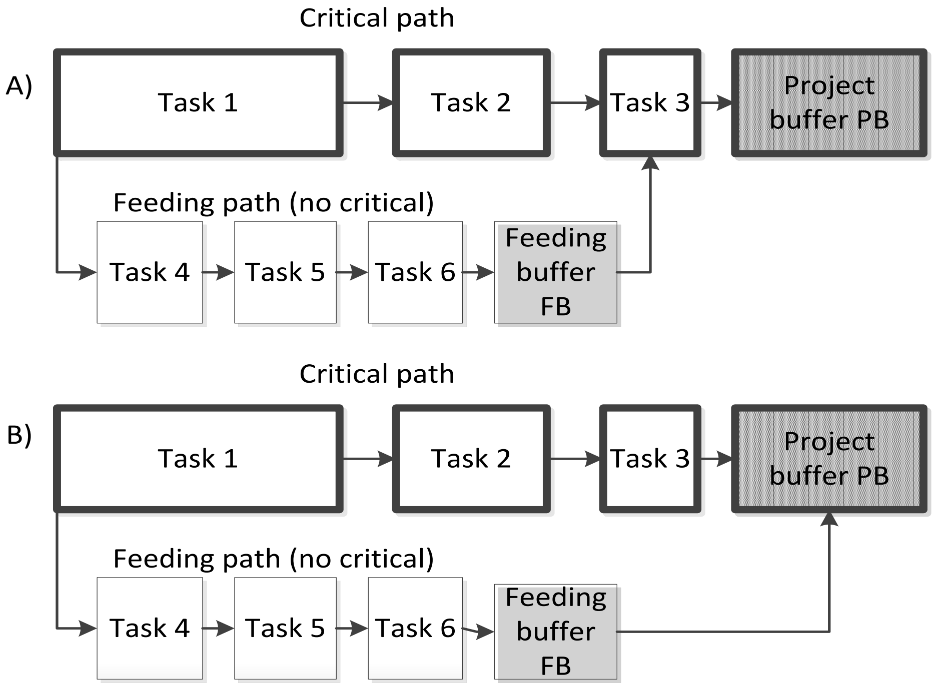

Figure 2. Scheme of feeding buffer (FB) location.
Source: personal elaboration

Summing up: project (PB) and feeding buffers (FB) should be located and included in the calculations in schedules made with application of the CC/BM method. They are introduced as fiction tasks with an appointed duration that do not engage any resources. Due to such structure of the relationship network, reduced task durations and accepted buffer times, the probability of fulfilling the completion date is at an appointed, relatively high level. The presence of buffers, through their observation, additionally allows controlling enterprise completion. When there is a threat that the planned completion dates will be delayed, it is possible to undertake appropriate preventive measures that will increase the work rate. Such actions are known as buffer management [Czarnigowska, Jaśkowski, Sobotka 2004].

This short account of the Goldratt method does not exhaust all its aspects that can be found in the literature [Goldratt 1997, Hejducki, Rogalska 2004, Leach 2000, Rand 2000, Raz, Barnes, Dvir 2003, Rogalska 2005, Steyn 2002]; here they have been presented in a way to ensure the presentation of the CC/BM method in civil engineering enterprises.

3 New concept of time buffers location

Due to the specific character and complexity of network schedules used in planning civil structures, practical application of the classical CC/BM method encounters significant problems. Schedules in civil engineering are commonly composed of over a hundred tasks, have different relationship types, numerous initial and/or final tasks, and the critical path is rarely a single sequence. Simple application of the guidelines of the CC/BM method to such schedules often leads to change in the course of the critical chain, thus breaks one of the basic principles of the method. The authors have drawn up a modification of the original Goldratt method by introducing additional buffer types [Pruszyński 2012]:

- buffer contributing to the project (PCB), which will save time float for the supply sequence so that it does not become a new critical chain (Figure 3). Its location is estimated on the critical path before the task, with which connects the discussed non-critical sequence. Its sizing is conducted following the same principle as for the project buffer, but taking into account the durations of tasks located on the critical path protected by the relevant contributing buffer (PB), i.e. lying between this buffer and the previous contributing buffer (or the beginning of the network).

The sizes of subsequent contributing buffers (PCB) reduce the time of the final buffer in the network, i.e. the new project buffer (PB') actually becomes one of the contributing buffers (PCB). However, the following principle should be fulfilled:

$$\sum_{i=1}^{n} PCB_i + PB' = PB \qquad (1)$$

where:

PCB_i – i-th buffer contributing to the project,

PB' – new project buffer,

PB – project buffer according to the assumptions of the critical chain,

n – number of buffers contributing to the project.

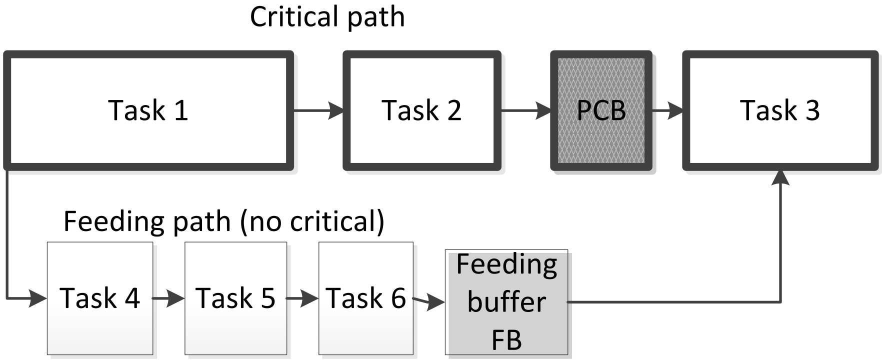

Figure 3. Scheme showing the location of the buffer contributing to the project (PCB).

Source: personal elaboration

- buffer reacting on the critical path (RCB) and buffer reacting on the non-critical path (RNB), applied in the case when the schedule contains sequences that have a common fragment (Figure 4). Figure 4a shows location of buffer reacting on the critical path, whereas Figure 4b – location of buffer reacting on the non-critical path. These buffers should be applied at the end of the common path at its junction. They are aimed at precluding repetition of calculations for tasks that are common for both sequences. Lack of these buffers will result in an erroneous result of determining the sizes of project (PB) and feeding buffers (FB) on relevant sequences.

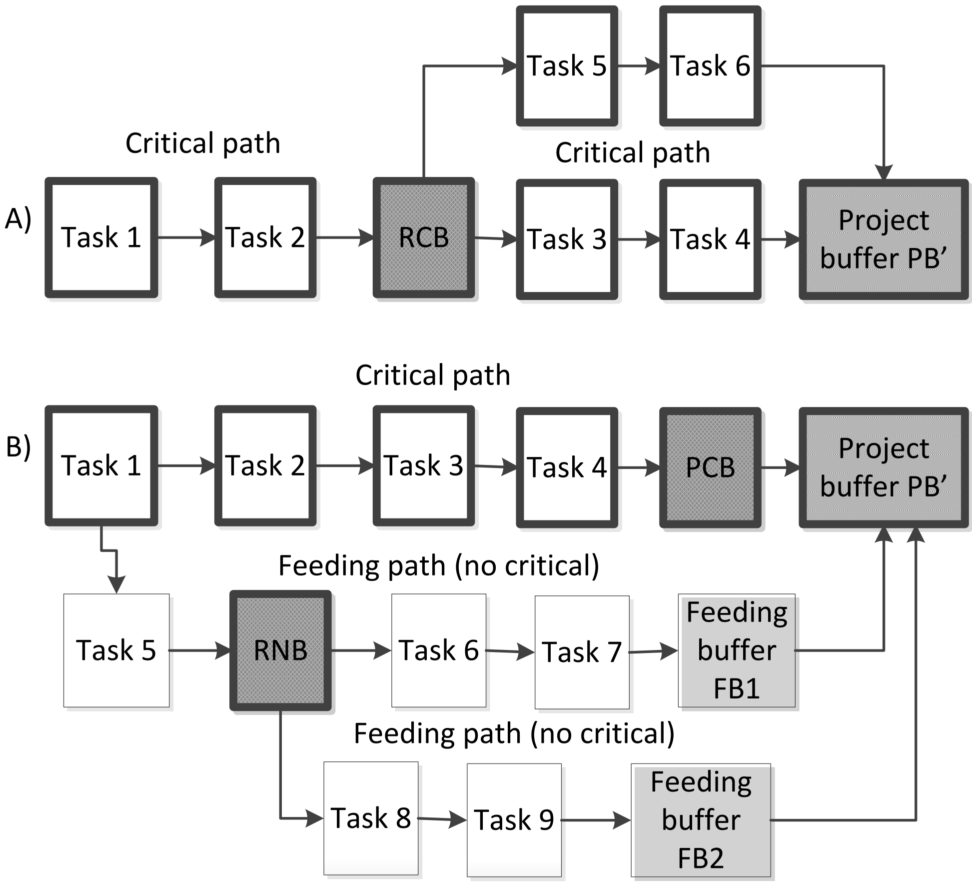

Figure 4. Scheme showing the localization of the buffer reacting on the critical path (RCB) and on the non-critical path (RNB).

Source: personal elaboration

- ending buffer (EB) is located at the ends of paths not connected later with the rest of the network and not terminated with other buffers (Figure 5), causing that the only termination of the network is a new project buffer. Its task is the time protection of sequence fragments – both on the critical path and beyond it – which have no other protection. Its size is determined similarly as in the case of other buffers.

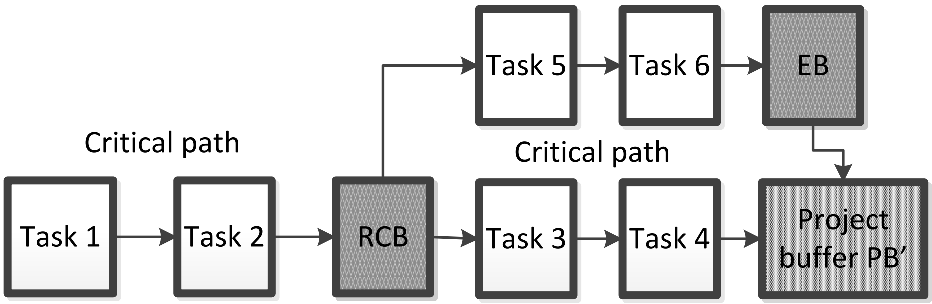

Figure 5. Scheme showing the localization of the ending buffer (EB).

Source: personal elaboration

Such location and accordingly determined sizes of the buffers result in reduction of the enterprise duration according to the assumptions of the CC/BM method at an unchanged position of the critical sequence.

4 Methods of buffer sizing

4.1. Type of duration distribution with regard to the method of buffer sizing

The founder of the assumptions in the critical chain method, E. Goldratt, did not draw up a detailed method of reducing task duration and the related buffer size. He just presented a general assumption that task durations may be reduced even to values ensuring a 50% certainty of fulfilling their duration, and half of the obtained reduction may be thus transferred to the buffers. The total time, by which the critical sequence was shortened, reduced by the value of the project buffer (lying on the path), is the reduction of the planned enterprise. Such simplification is practical when calculations are done but difficult to justify from the theoretical point of view. Therefore, several proposals have been presented where the buffer size depends on the value of standard deviations of the tasks, at the end of which the buffer is located [Milian 2006, Stępień 2012]. However, in this case calculation of the duration tasks and buffer sizes for each task will require positing two basic assumptions. The first one refers to the type of duration distribution of a single task, and the second – the type of distribution of the completion date (or the date of buffer completion).

The type of distribution of a single task in the completion of a specific civil structure is difficult to determine precisely. In literature and engineering practice the following distributions are usually assumed: beta, beta-PERT, normal, lognormal, triangular, rarely uniform or exponential. Each of these distributions has its own characteristics and resultant parameters, mainly variance and characteristic quantiles, and methods of their estimation. Ways of their determination and the obtained values will thus influence the distribution parameters of completion dates of the tasks sequence.

Considering the distribution of the final task duration (or generalizing, any sequence of subsequent tasks), we refer to the central (boundary) statement and assume that the distribution of the sum of independent tasks (with any distribution) approaches a normal distribution with the increase of the task number. It should be pointed out that even when the number of the investigated variables is moderately high, then if none of the variables is dominant and the variables are not highly dependent, the distribution of their sum will be close to a normal distribution [Benjamin, Cornell 1977]. At low number of variables, distribution of their sum will be close rather to t-Student distribution than to normal distribution.

Considering the problem of buffer size, we refer to the fact that the variability of a sum is always smaller than the sum of variability's of particular components. This is a very important property of distributions, due to which it is possible to reduce particular tasks in the sequence (thus decrease the probabilities of their fulfilment), retaining probability of the completion date at a specific level. Naturally, this procedure can be done in a certain range of task durations, depending mainly on the probability of fulfilling task duration, with which the initial (non-reduced) durations of particular tasks were determined.

4.2 Method of task duration reduction

Calculation of project buffer duration requires determining the course of the critical chain in the analysed network. Next, the start and finish date of the sequence should be exchanged for subsequent days of the enterprise. Thus, numerical values of start and finish in the critical chain are obtained. Its duration should be calculated and the difference between the finish and start of this sequence should be determined.

In the subsequent calculation step, the duration of tasks lying on the reduced path of the critical chain are shortened. If analysis of the risk of the planned enterprise was carried out earlier, reference to its results can be made during task shortening and the determined risk coefficients can be accepted as the reduction indicator [Połoński 2013]. Otherwise, the method of percentage task duration reduction is usually used. Percentage reduction may be identical for all tasks or may be different, e.g. for critical and non-critical tasks. It should be recalled that reduction of each task must ensure the technological-organizational possibilities of its fulfilment.

Using reduction values of particular tasks, the project buffer (PB) value is determined. Difference between the duration of the initial and reduced critical sequence should be calculated. In the end, percentage reduction of the project duration should be made and the value of the project buffer should be determined. Sizes of all other buffers should be calculated following the same procedure.

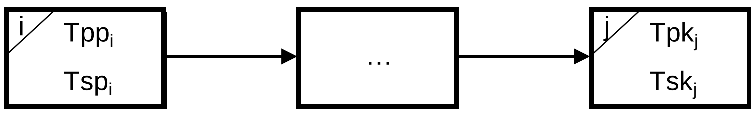

Figure 6. Calculation scheme for determining sequence duration in a PDM diagram.

Source: personal elaboration

Durations of particular buffers are calculated according to the following formula:

- In a PDM diagram (Figure 6), used e.g. in the MS Project software, the formula can be written down as follows:

$$WB_{ij} = P \cdot [(Tpk_j - Tpp_i) - (Tsk_j - Tsp_i)], \tag{2}$$

where:

WB_{ij} – size of the time buffer protecting the sequence between tasks *i-j*,

P – percentage reduction of the sequence duration time,

i – number of task starting the sequence,

j – number of task finishing the sequence,

Tpp_i – date of latest start of task *i* prior to reduction,

Tpk_j – date of latest finish of task *j* prior to reduction,

Tsp_i – date of latest start of task *i* after reduction,

Tsk_j – date of latest finish of task *j* after reduction.

- In an ADM diagram (Figure 7), the formula can be written down as follows:

$$WB_{ij} = P \cdot [(NP_j - NP_i) - (NP'_j - NP'_i)], \tag{3}$$

where:

WB_{ij} – size of the time buffer protecting the sequence between tasks *i-j*,

P – percentage reduction of sequence duration,

i – number of task, during which the analysed sequence starts,

j – number of task, during which the analysed sequence finishes,

$t1, ..., t_n$ – task durations prior to reduction,

$t'1, ..., t'_n$ – task durations after reduction,

NP_i – latest start date for task *i* prior to reduction,

NP_j – latest start date for task *j* prior to reduction,

NP'_i – latest start date for task *i* after reduction,

NP'_j – latest start date for task *j* after reduction.

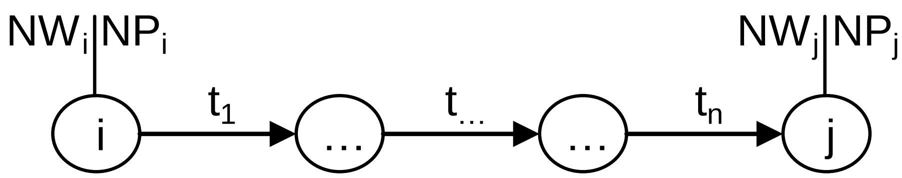

Figure 7. Calculation scheme for sequence duration in an ADM diagram.

Source: personal elaboration

As shown earlier, when applying percentage reduction of task duration in buffer sizing, it is most convenient to use numerical values of the start and finish for the analysed tasks instead of dates.

4.3 <u>Method resulting from assuming a normal distribution of task durations</u>

To calculate buffer sizes based on normal distribution of task durations, one should assume that task durations presupposed by the contractor in the schedule have been determined with a probability of e.g. 0.9 ($t_{0.9}$), and estimate task durations with a different probability e.g. 0.5 ($t_{0.5}$). Based on the presupposed estimates of task durations and assuming the distribution of a single task duration, further calculation can be made. In this case, normal distribution was presupposed as the distribution type for a single task duration.

For further simplification of buffer size calculations, it has been assumed that the relationship network in the analysed schedule contains only finish–start (F-S) relationships between the tasks with possible positive or negative values of lag in these relationships.

Based on the assumed two quantiles of the duration of all tasks and knowing that they refer to normal distribution, standard deviation of task durations (or delays in the relationship) can be calculated according to the following formula:

$$\delta = \frac{t_{0.9} - t_{0.5}}{U_{0.9}} \tag{4}$$

where:

$U_{0.9}$ – inverse of cumulated, standard normal distribution

Based on the calculated standard deviation and the assumed two quantiles of task durations, the quantile can be calculated with an assumed optional probability. If F-S relations occur with delays or accelerations between the tasks, the reduced durations of these relationships should be calculated similarly as for the tasks.

The next step in the calculations is determining buffer location, number and sizes. In analysis without considering resources, the critical path is the critical chain in the schedule. A project buffer is located at its end; its aim is to protect the fulfilment of the enterprise completion date. Feeding buffers are located at the ends of non-critical sequences that connect with the critical sequence. They protect tasks in the critical sequence prior to the change of the location of this sequence. If needed (e.g. when during changes of task durations and introduction of project and feeding buffers, the position of the critical chain underwent changes), the location and sizes of the remaining buffers should also be determined.

The completion date of the entire sequence of tasks with an earlier assumed probability should be calculated from the cumulative normal distribution function based on parameters of this distribution:

$$m_T = \sum_{i=1}^{k} t_{0.5}, \tag{5}$$

$$\delta_T = \sqrt{\sum_{i=1}^{k} \delta_i}, \tag{6}$$

where:

δ_i – standard deviation of task *i* duration,

k – number of tasks in the analysed sequence.

Buffer sizes are calculated as the difference between the completion date for the entire task sequence protected by the buffer at an assumed probability level, and the sum of task durations assumed for this probability.

It should also be noted that when at least one contributing buffer (PCB) occurs beside the project buffer in the critical sequence, the sum of durations of these buffers calculated according to formula (6) will be higher that the primary, single project buffer (PB) (this results from the statistical principle that the sum of standard deviations of variables is larger than the standard deviations of these sums). In consequence, this property causes that the enterprise completion date is slightly longer than calculated earlier, and the probability level of its fulfilling is slightly higher than earlier assumed. In order to avert this, values of the calculated contributing buffers (PCB) and the modified project buffer (PB’) should be reduced to a level assuring that their sum will represent the original value of an unmodified project buffer (PB).

4.4 Method resulting from assuming a lognormal distribution of task duration

The origin of this distribution is accepting an assumption that the shape of the distribution and its characteristic parameters undergo a multiplicative mechanism, thus result from the product of numerous random factors influencing the analysed variable. According to this assumption, a non-symmetrical, rightward skewed distribution is obtained, in which the variable has a lognormal distribution, whereas its logarithm has a normal distribution. The presented distribution characteristics well fits the distribution of a single task duration, influenced by an abundant group of random factors, whereas the right-skewness of the distribution well describes the assumed distribution of single task duration, because practice shows that its delay occurs more commonly than its acceleration.

Buffer sizing requires knowledge of two parameters for each task: average value t_m and standard deviation δ_t. The average value of task duration in a lognormal distribution is shown by the following formula [Benjamin, Cornell 1977] :

$$t_m = t_{0.5} \cdot \exp\left(\frac{\delta^2_{\ln t}}{2} \right) \tag{7}$$

where:
t_m – average value of task duration,
$t_{0.5}$ – quantile of 0.5 of task duration,
$\delta^2_{\ln t}$ – square of standard deviation of task duration logarithm.

In order to determine the $\delta^2_{\ln t}$ parameter is used a formula for standard deviation in a normal distribution δ_t calculated on the basis of two quantiles. In this case it was assumed that the second known quantile is quantile 0.9, however a similar formula can be derived for any given quantile:

$$\delta_t = \frac{t_{0.9} - t_{0.5}}{U_{0.9}} \tag{8}$$

where:
$t_{0.9}$ – quantile of 0.9 of task duration,
$t_{0,5}$ – quantile of 0.5 of task duration,
$U_{0.9}$ – value of cumulative distribution function for standard normal distribution N(0.1) for probability 0.9.

Based on formula (8) and assumptions of lognormal distribution it can be seen that:

$$\delta_{\ln t} = \frac{\ln(t_{0.9}) - \ln(t_{0.5})}{U_{0.9}} \tag{9}$$

and the wanted average value t_m is:

$$t_m = t_{0.5} \cdot \exp\left(\frac{\ln^2\left(\frac{t_{0.9}}{t_{0.5}} \right)}{2 \cdot U^2_{0.9}} \right) \tag{10}$$

where:
t_m – average time duration with lognormal distribution; the remaining symbols are explained in formulas (7) and (8).

Time buffer sizing requires also knowledge of standard deviation of a variable with lognormal distribution. It can be determined from the following formula [Benjamin, Cornell 1977]:

$$\delta^2{}_t = t^2{}_{0.5} \cdot \exp(2 \cdot \delta^2{}_{\ln t}) - t^2{}_{0.5} \cdot \exp(\delta^2{}_{\ln t}) \tag{11}$$

which after transformation leads to the formula:

$$\delta^2{}_t = t^2{}_{0.5} \cdot (\exp(\delta^2{}_{\ln t})) \cdot (\exp(\delta^2{}_{\ln t}) - 1) \tag{12}$$

where:
$\delta^2{}_t$ – square of standard deviation of task duration with lognormal distribution; the remaining symbols are explained in formulas (7) and (8).

Knowing that the completion date of a task sequence has a normal distribution N (m_T, δ_T) with parameters:

$$m_T = \sum_{i=1}^{k} t_{mi} \quad , \quad \delta_T = \sqrt{\sum_{i=1}^{k} \delta_{ti}{}^2} \tag{13}$$

where:
k – number of tasks in the analysed sequence,

the completion time of this sequence at specific probability level can be determined. Therefore, e.g. assuming the probability level at 0.9, the completion date of a task sequence with this probability will be:

$$T_{0.9} = \sum_{i=1}^{k} t_{mi} + U_{0.9} \cdot \sqrt{\sum_{i=1}^{k} \delta^2{}_{ti}} \tag{14}$$

where:
$T_{0.9}$ – completion date of sequence of k tasks with probability at 0.9,
t_{mi} – average duration of task i,
$\delta^2{}_{ti}$ – square of standard deviation of *i-th* task duration,
$U_{0.9}$ – value of cumulative distribution function for standard normal distribution N(0.1) for probability at 0.9,
k – number of tasks in sequence.

At a low number of tasks in the sequence (k), the value of the standardized variable $U_{0.9}$ should rather be read from the t-Student distribution (for k-1 number of freedom degrees), because this distribution better describes the distribution of the analysed parameter, in this case the total duration time of the analysed tasks.

The CC/BM method assumes task shortening. A correct formula has to be derived when the duration times of particular tasks with a specific probability (determined quantiles) are to be calculated. In the case of normal distribution, the quantile of task duration with probability p can be calculated from the formula:

$$t_p = t_{0.5} + \delta_t \cdot U_p \tag{15}$$

where:
t_p – quantile p of duration time,
δ_t – standard deviation of duration time,
U_p – value of cumulative distribution function for standard normal distribution N(0.1) for probability p.

Remembering that in a lognormal distribution, logarithms of variables have normal distribution, and assuming that quantiles of task duration 0.5 and 0.9 have been used to calculate the standard deviation δ_t, using formula (8) and the logarithm definition we can calculate:

$$t_p = \exp(U_p \cdot \delta_t + t_{0.5}) = \exp\left[U_p \cdot \frac{1}{U_{0.9}} \cdot \ln\left(\frac{t_{0.9}}{t_{0.5}}\right) + \ln(t_{0.5})\right] \tag{16}$$

Assuming that $A = \dfrac{U_p}{U_{0.9}}$ and transforming formula (16), we obtain:

$$t_p = \exp\left(A \cdot \left\langle \ln(t_{0.9}) - \ln(t_{0.5}) \right\rangle + \ln(t_{0.5})\right) \tag{17}$$

where:
t_p – quantile p of task duration with lognormal distribution,
U_p – value of cumulative distribution function for standard normal distribution N(0.1) for probability p.

5 Algorithm for determining enterprise completion time with consideration of buffers

Application of the CC/BM method requires:

- drawing up a construction of the relationship network for the planned enterprise, determining task duration times at a high probability level of their fulfilment, e.g. 0.9, and determining the course of the critical chain,
- introduction of new, fiction tasks (time buffers) into the network,
- reducing task durations to a lowered probability level of their fulfilment, e.g. 0.5 or 0.7, and transferring part of the reductions to the buffers,
- determining the distribution of the new completion date for the planned enterprise and calculating the enterprise completion date with a specific probability (e.g. 0.9).

It is also assumed that the introduction of time buffers and changed (shorter) task durations does not cause changes in the course of the critical chain. Accomplishment of the presented assumptions requires practical solution of the following issues:

- determining the number, types and location of time buffers in the relationship network,
- determining how much the durations of particular tasks have to be reduced,
- checking if the conducted calculations have not caused changes in the course of the critical chain,
- sizing of all buffers in relation to reduced task durations,
- assuming distribution types of the completion date for the enterprise and particular buffers, and determining their parameters,
- calculating the enterprise completion date with an assumed probability.

This list shows that in the case of a complex structure of the relationship network, solving particular issues requires a series of activities, whereas the assumptions accepted during their execution and exact calculated values will influence the final result. When accepting a specific variant of the CC/BM method, the practical side of the calculations and their introduction in wide engineering practice should be remembered.

Figure 8 shows a scheme, which sets out the general methodology of preparing a network schedule to apply the critical chain method and to conduct calculations. It points out the problems that can be met with during their execution in real conditions of construction works and indicates the possible solutions. The following scheme (Figure 9) illustrates the method of determining the location of particular buffers and their sizing.

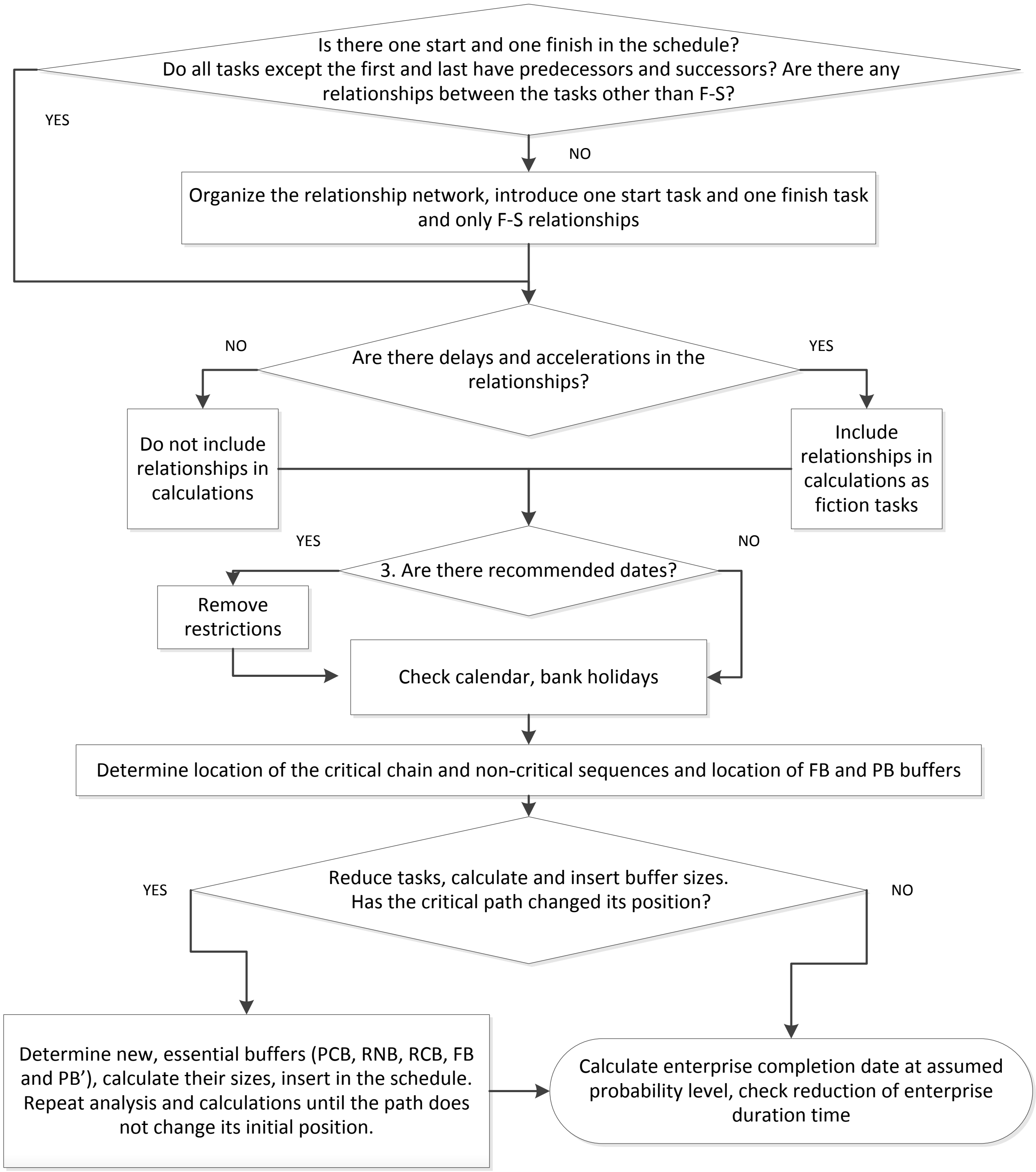

Figure 8. Scheme of measures during determination of time buffer sizes and enterprise completion date.

Source: personal elaboration

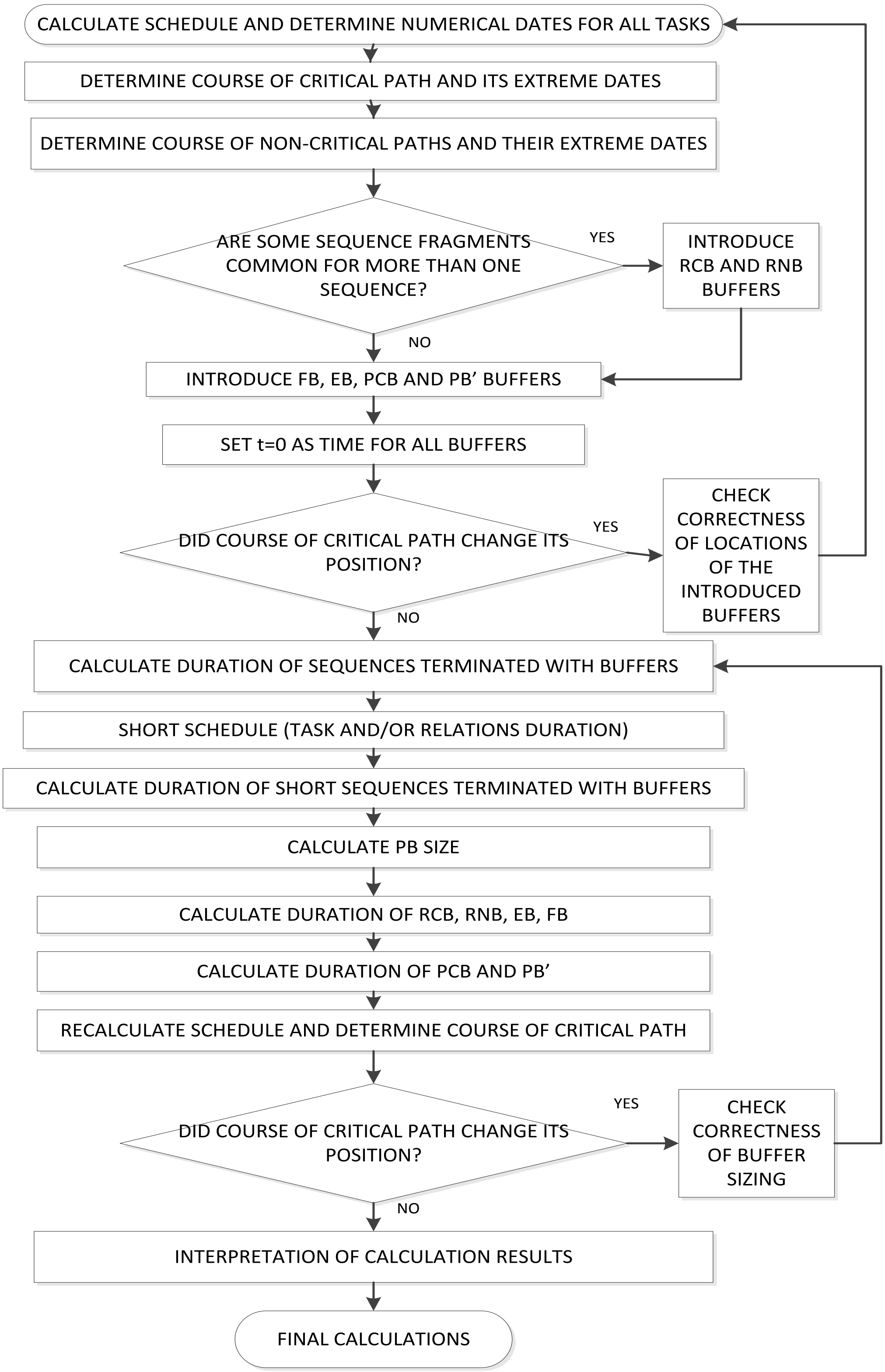

Figure 9. Block scheme of activities during application of a modified critical chain method according to the authors' proposal.

Source: personal elaboration

6 Example calculation

Below are presented results of calculations based on a CPM schedule for a civil structure according to the method of buffer location and sizing proposed by the authors. The investment with CPM schedule in the presented example is the extension and modernization of a municipal sewage plant in Baranów [Szulc 2008]. Initially, the CPM schedule had 66 tasks. However, due to the possibility of presenting the calculated data, it was aggregated and shortened to a small network fragment, selected to represent a work fragment most complex with regard to technology and organization. Thus, it was restricted to 41 tasks and three control points (without buffers) (Figure 10). Five buffers were introduced to the network hierarchy: 3 feeding buffers (FB1, FB2, FB3), one contributing buffer on the critical sequence (PCB1) and one project buffer (PB) at the end of the critical chain. The number and types of buffers have been selected based on the hierarchy of the analysed relationship network. In the figure the thick black frames refer to critical chain tasks. The relationship network was drawn up in MS Project. Table 1 presents tasks from this network with basic data needed for further analysis. The enterprise completion date at time $t_{0.9}$ was 229 days, and at time $t_{0.5}$ 136 days.

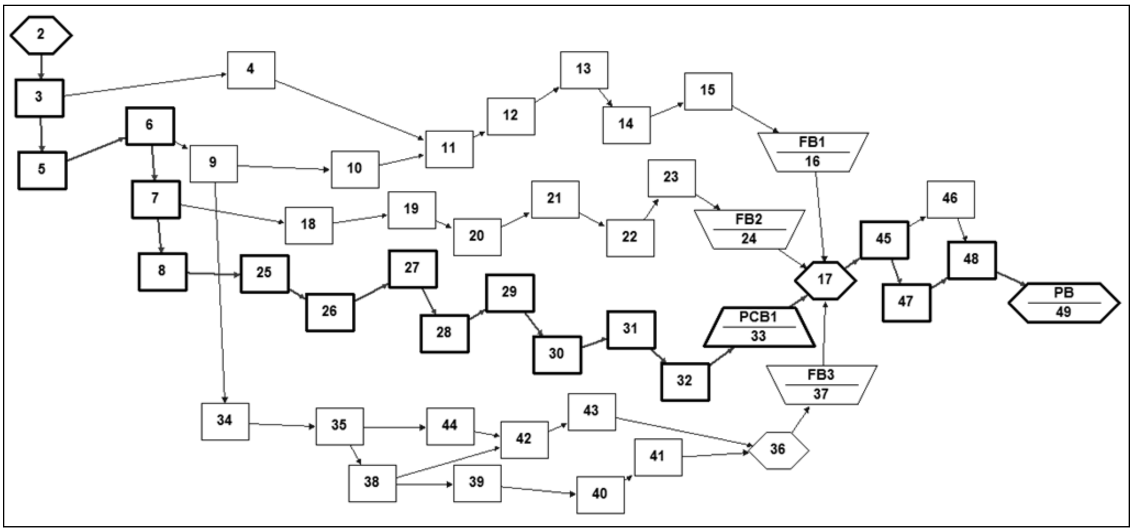

Figure 10. Selected network fragment with course of critical chain and buffer location.

Source: personal elaboration

In the presented example it has been assumed that the initially accepted task durations ($t_{0.9}$) were determined by the contractor with probability at 0.9 and have lognormal distribution. Next, task durations $t_{0.5}$ were estimated, whose probability is at 0.5 (Table 1). Using two task duration quantiles, average task durations were calculated with formula (10), whereas squares of standard deviation of these durations were determined using formula (12). Knowing quantiles $t_{0.5}$ and $t_{0.9}$ of task durations, any other quantiles were calculated using formula (17). Two quantiles with probability at 0.55 and 0.70 were applied in further discussions.

The next calculation step was determining completion dates for the enterprise and for sequences protected by particular buffers with an assumed probability. The probability of fulfilling the dates assumed in the example was at 0.9. The final distribution of the task sequence was normal distribution N (m_T, δ_T) with parameters determined from formula (7) or t-Student distribution with the number of freedom degrees at *k*-1, where k is the number of tasks with duration above zero in the analysed sequence. Table 2 presents task sequences protected by particular buffers, and Table 3 – examples of calculations for buffer FB1.

All calculations were made with use of MS Excel. A template was prepared in which task durations $t_{0.5}$ and $t_{0.9}$ were inserted; based on them the buffer size was calculated and all remaining values were calculated automatically. Buffer duration was calculated as the difference between the completion date of the entire sequence with probability at 0.9 and the sum of task duration accepted for a given quantile.

Table 1. Summary of tasks in the selected fragment of relationship network.

ID	$t_{0.9}$	$t_{0.5}$	predecessors		ID	$t_{0.9}$	$t_{0.5}$	predecessors		ID	$t_{0.9}$	$t_{0.5}$	predecessors
total	229	136			17	0	0	16; 37; 33; 24		33	0	0	32
2	0	0			18	5	3	7		34	10	6	9
3	10	6	2		19	45	24	18		35	4	2	34
4	50	26	3		20	10	6	19		36	0	0	43; 41
5	20	11	3		21	10	6	20		37	0	0	36
6	28	15	5		22	2	1	21		38	3	2	35
7	28	15	6		23	1	1	22		39	3	2	38
8	20	11	7		24	0	0	23		40	3	2	39
9	5	3	6		25	10	6	8		41	2	1	40
10	45	24	9		26	5	3	25		42	3	2	44; 38
11	10	6	10; 4		27	20	11	26		43	2	1	42
12	10	6	11		28	20	11	27		44	2	1	35
13	2	1	12		29	4	2	28		45	7	4	17
14	1	1	13		30	7	4	29		46	10	6	45
15	10	6	14		31	5	3	30		47	15	8	45
16	0	0	15		32	20	11	31		48	10	6	46; 47
										49	0	0	48

Source: personal calculations.

Table 2. Summary of buffers and task sequences protected by these buffers

No.	Buffers	Protected task sequence ID	Number of tasks in sequence	Number of tasks in sequence t > 0
1	FB1	9;10;11;12;13;14;15	7	7
2	FB2	18;19;20;21;22;23	6	6
3	FB3	34;35;38;39;40;41;36	7	6
4	PCB1	2;3;5;6;7;8;25;26;27;28;29;30;31;32	14	13
5	PB	17;45;47;48	4	3

Source: personal calculations.

Table 3. Example of calculation of buffer FB1 size for protected duration of sequence with normal and t-Student distribution and two task duration quantiles of this sequence: 0.55 and 0.70.

Task ID	$t_{0.5}$	$t_{0.9}$	t_m	δ_t^2	$t_{0.55}$	$t_{0.70}$
9	3	5	3.248	1.817	3.2	3.7
10	24	45	27.068	199.293	25.5	31.0
11	6	10	6.496	7.267	6.3	7.4
12	6	10	6.496	7.267	6.3	7.4
13	1	2	1.158	0.455	1.1	1.3
14	1	1	1.000	0.000	1.0	1.0
15	6	10	6.496	7.267	6.3	7.4
Sum	*47*	*83*	*51.962*	*223.365*	*49.7*	*59.2*
Buffer time at task durations			$t_{0.50}$	t_m	$t_{0.55}$	$t_{0.70}$
Normal distribution			24.1	19.2	21.4	11.9
t-Student distribution			34.0	29.0	31.3	21.8
Completion date with probability at 0.9 for normal distribution						71.1
Completion date with probability at 0.9 for t-Student distribution						81.0

Source: personal calculations.

After conducting similar calculations for all buffers, calculations of the relationship network were made in MS Project. In each analysed variant, after approximating to whole days, were inserted task durations to the analysed quantile, buffer times corresponding to this quantile, and assumed completion time distribution: normal or t-Student.

Final calculations are presented in Table 4. Four calculation variants of the whole schedule are given for four different task durations: average t_m and quantiles 0.50, 0.55 and 0.70. Three results are given for each variant: a – assuming that all other buffers equal zero (for comparison), b – assuming that completion date of a task sequence protected by the buffer has normal distribution, and c – assuming that completion date of a task sequence protected by the buffer has t-Student distribution. It is clear that with increased duration time, buffer sizes decrease, which depending on the accepted quantile of task duration allows the project contractor to choose a more or less aggressive schedule variant. Depending on the selected calculation variant, the enterprise completion time may vary from 168 to 192 days (at 229 days without buffer application). The probability of fulfilling the enterprise completion date was also determined based on two distributions: normal and t-Student. It is notable that more optimistic results are obtained based on normal distribution (0.912 to 0.995), whereas less optimistic results are acquired in t-Student distribution (0.803 to 0.978).

Table 4. Enterprise completion dates, probabilities of their fulfilment and percentage reduction for various duration times and two distribution types of sequence duration: normal and t-Student.

No	**Quantile of task duration**	**Buffer times**					**Distribution of sequence completion date**	**Enterprise completion date**	**% reduction of enterprise duration**	**Probability of completion date according to distribution**	
		BZ1	**BZ2**	**BWP1**	**BZ3**	**BP’**				**normal**	**t-Student**
1a	t_m	0	0	0	0	0		137	40	0.416	0.000
1b	t_m	19	19	24	4	7	Normal	168	27	0.912	0.803
1c	t_m	29	30	34	7	17	T-Student	188	18	0.991	0.968
2a	$t_{0.5}$	0	0	0	0	0		127	45	0.237	0.000
2b	$t_{0.5}$	24	23	37	6	9	Normal	173	24	0.946	0.870
2c	$t_{0.5}$	34	34	46	8	19	T-Student	192	16	0.995	0.978
3a	$t_{0.55}$	0	0	0	0	0		134	41	0.358	0.000
3b	$t_{0.55}$	21	21	30	5	8	Normal	172	25	0.940	0.859
3c	$t_{0.55}$	31	32	40	7	18	T-Student	192	16	0.995	0.978
4a	$t_{0.70}$	0	0	0	0	0		157	31	0.787	0.562
4b	$t_{0.70}$	12	12	7	2	5	Normal	169	26	0.919	0.818
4c	$t_{0.70}$	22	23	16	5	14	T-Student	187	18	0.990	0.964

Source: personal calculations.

7 Conclusions

The authors’ experience with attempts of locating time buffers in large and complex CPM schedules of construction objects prepared according to the Goldratt method indicates that the application of only feeding buffers and a project buffer leads to changes of the critical path course. The original method of applying new types of buffers, which is presented herein, solves this problem. Particularly crucial is the introduction of contributing buffers located on the critical path. The paper describes the method of their location with regard to the course of non-critical paths connected with the critical path. Introduction of new buffers required working out a method that would calculate their duration. A method of such calculations is proposed, taking into account two types of duration distribution in a single activity: normal distribution and lognormal distribution. The overall actions needed to introduce all indispensable buffers into the schedule according to the method proposed by the authors have been illustrated in a block diagram. The presented example shows the practical application of the proposed method along with the required calculations and their results.

As already mentioned, CPM schedules are characterized by high complexity and various schemes of critical and non-critical paths. Therefore, further studies on the practical application of time buffers and the verification of the proposed solutions are required. Particular attention should be drawn on the correlation of buffer sizes with the number of protected activities, variability of duration times and methods of distinguishing time buffers in the case of other relationships between the activities other than finish-to-start relations.

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